EE 330
Lecture 15

Devices in Semiconductor Processes

- Diodes
- Analysis of Nonlinear Circuits

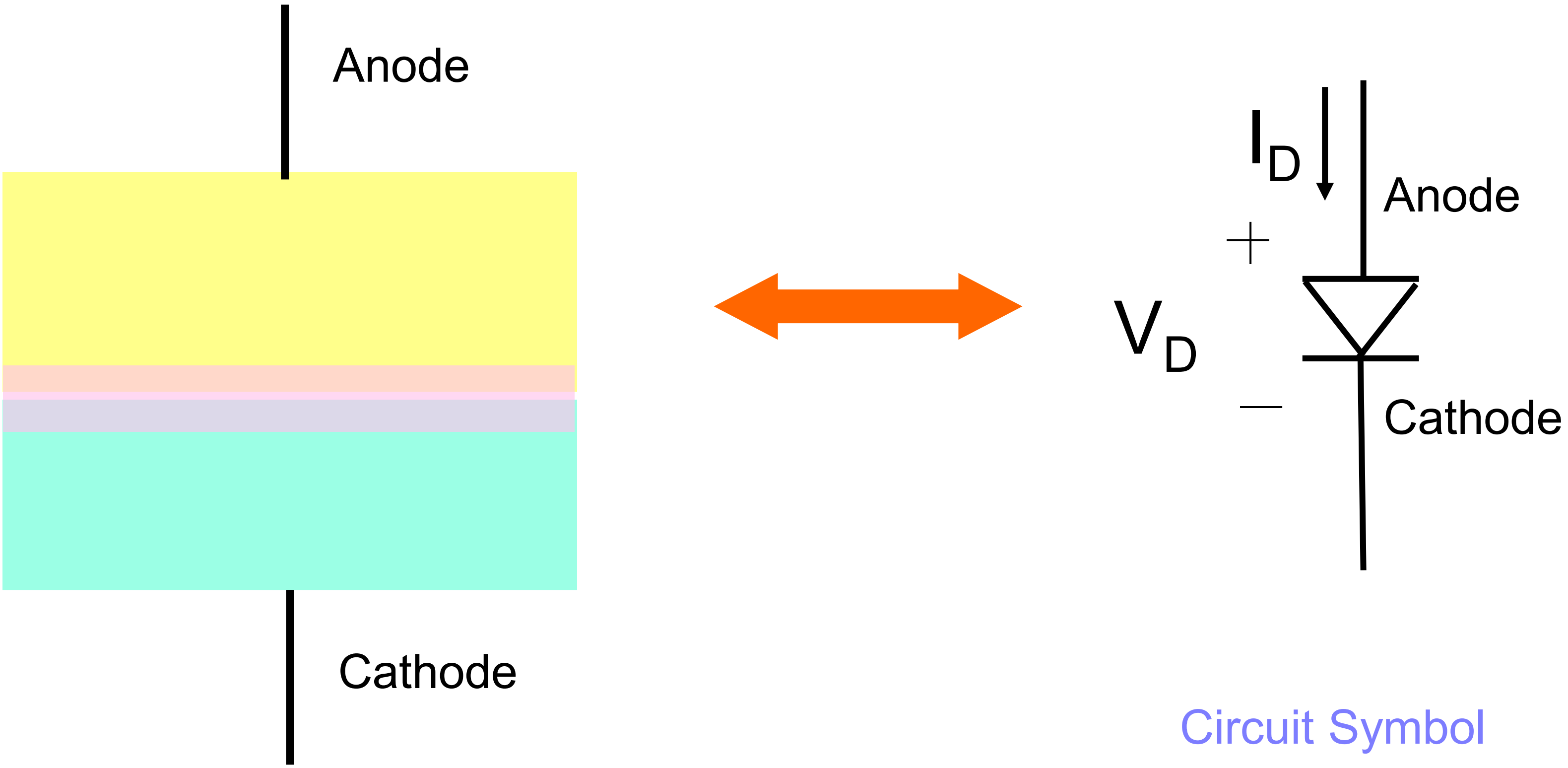
Spring 2024 Exam Schedule

Exam 1	Friday Feb 16
Exam 2	Friday March 8
Exam 3	Friday April 19

Final Exam Tuesday May 7 7:30 AM - 9:30 AM

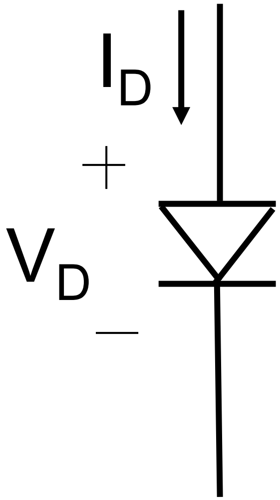
Review from last lecture

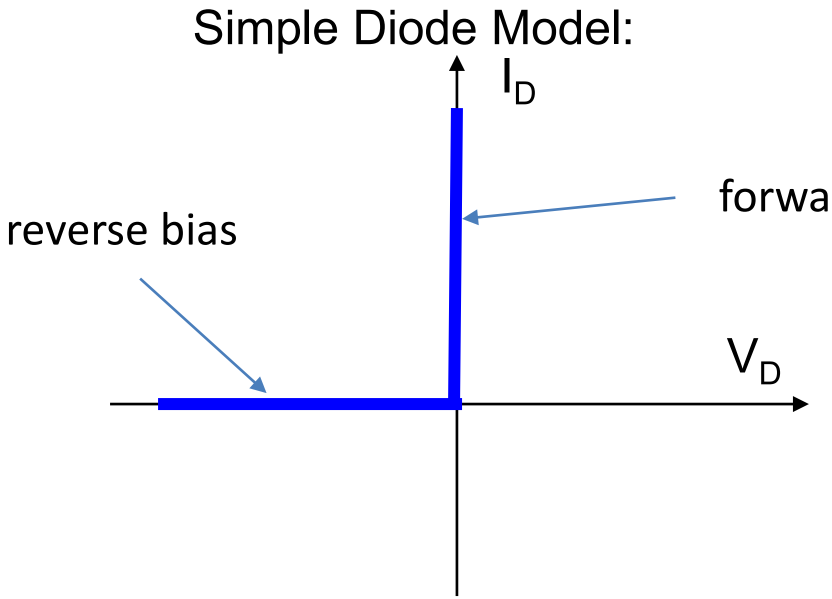
pn Junctions



Circuit Symbol

pn Junctions



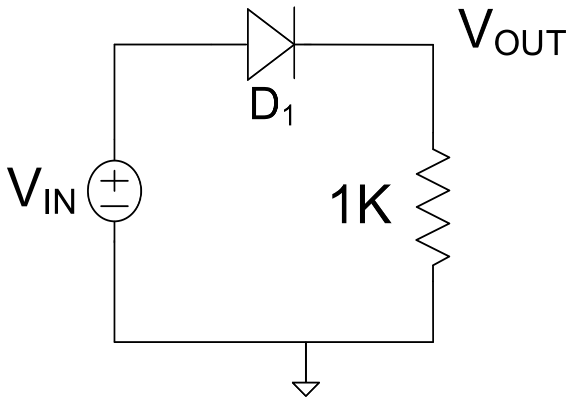


- This is a piecewise model

- pn junction serves as a "rectifier" passing current in one direction and blocking it in the other direction

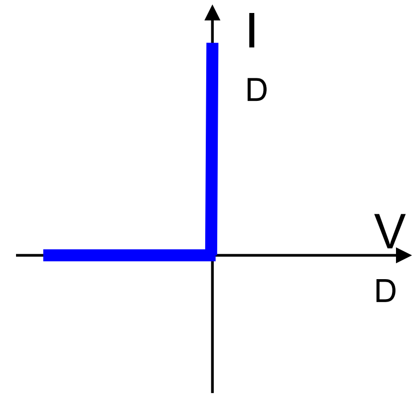
Review from last lecture

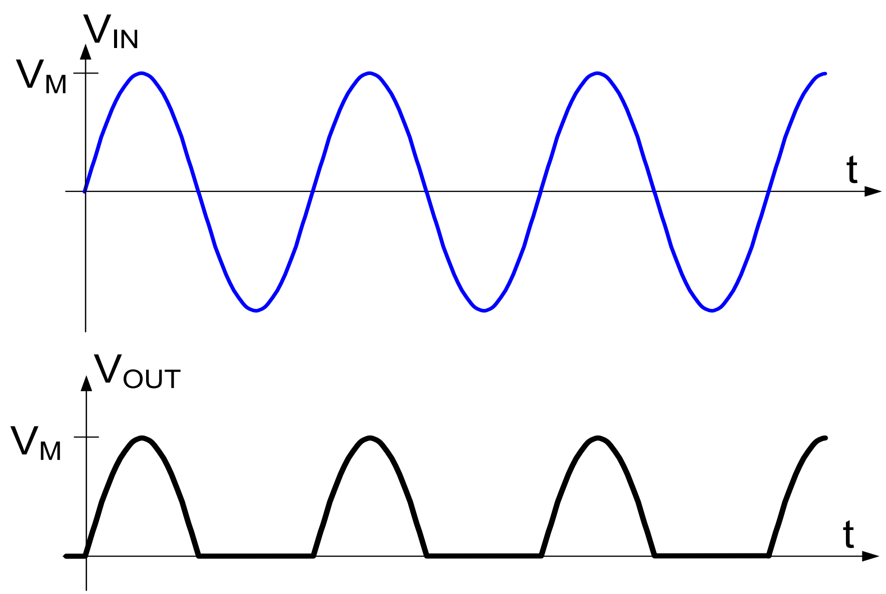
Rectifier Application:



$V_{IN}=V_M \sin\omega t$

Simple Diode Model:



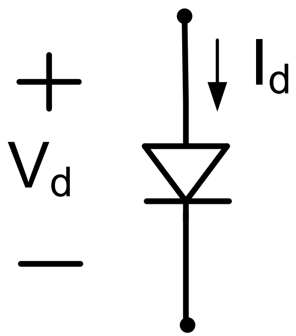


Analysis based upon “passing current” in one direction and “ blocking current” in the other direction

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:



I_S in the 10fA to 100fA range

I_S proportional to junction area

$$V_t = \frac{kT}{q}$$

k= 1.380 64852 × $10^{-23}$$JK^{-1}$

Diode Equation

$$I_D = I_S\left(e^{\frac{V_d}{nV_t}} - 1\right)$$

I_S and n are model parameters

What is V_t at room temp?

V_t is about 26mV at room temp

q = −1.60217662×10^{-19} C

k/q=8.62× 10^{-5} VK^{-1}

n typically about 1

Diode equation due to William Shockley, inventor of BJT

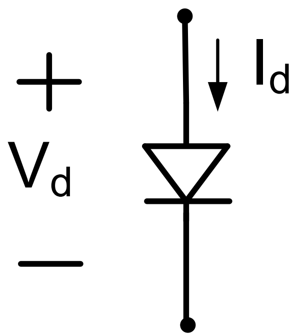
In 1919, [William Henry Eccles] coined the term ***diode***

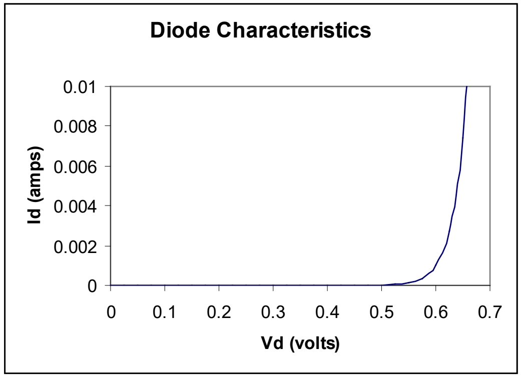
In 1940, Russell Ohl "stumbled upon" the p-n junction diode

I-V characteristics of pn junction

(signal or rectifier diode)

Improved Diode Model:





I_S in 10fA -100fA range (for signal diodes)

n typically about 1

$$V_t = \frac{kT}{q}$$

$k/q = 8.62 \times 10^{-5}\,VK^{-1}$

V_t is about 26mV at room temp

Diode Equation $$I_D = I_S\left(e^{\frac{V_d}{nV_t}} - 1\right)$$

(not a piecewise model !)

Simplification of Diode Equation:

Under reverse bias ($V_d<0$), $I_D \cong -I_S$

Under forward bias ($V_d>0$), $I_D = I_S e^{\frac{V_d}{nV_t}}$

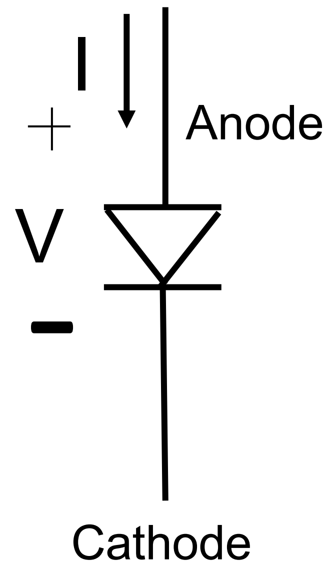
Simplification essentially identical model except for V_d very close to 0

Diode Equation or forward bias simplification are unwieldy to work with analytically

pn Junctions

Diode Equation: (simplification)

$$I = \begin{cases} I_S A e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ -I_S & V < 0 \quad \text{reverse bias} \end{cases}$$



Diode Equation: (further simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ 0 & V < 0 \quad \text{reverse bias} \end{cases}$$

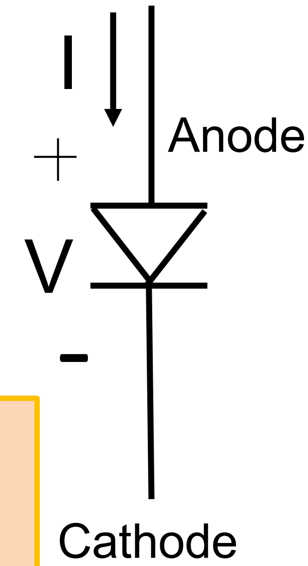
$I_S = J_S A$

{J_S} is model parameter (or I_S is a model parameter if A is fixed)
{A} is design parameter , A is the cross-sectional area of the junction

Slight discontinuity at V=0 in these models (which doesn't exist in real diodes) but of no consequence unless V is very close to 0

I_S is often given in data sheets and model files

Diode Model Summary



Ideal Diode Model

$V_D = 0$	$I_D > 0$	forward bias
$I_D = 0$	$V_D < 0$	reverse bias

$I_S = J_S A$

Diode Equation

$$I_D = I_S\left(e^{\frac{V_d}{nV_t}} - 1\right)$$

Diode Equation:
(simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ -I_S & V < 0 \quad \text{reverse bias} \end{cases}$$

Diode Equation:
(further simplification)

$$I = \begin{cases} I_S e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ 0 & V < 0 \quad \text{reverse bias} \end{cases}$$

Little difference in these models, if any, in most applications. Typically, any referred to as the Diode Equation

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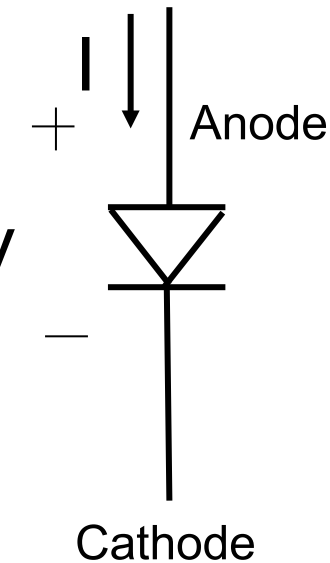
Diode Equation:
(further simplification)

$$I = \begin{cases} J_S A e^{\frac{V}{nV_T}} & V > 0 \quad \text{forward bias} \\ 0 & V < 0 \quad \text{reverse bias} \end{cases}$$

$I_S = J_S A$



J_S (or I_S) is strongly temperature dependent

With n=1, for V>0,

$$J_S = J_{SX} T^m e^{\frac{-V_{G0}}{V_t}}$$

{J_{SX}, m,n} are model parameters
{A} is a design parameter
{T, V_{G0}, k/q} are environmental parameters and physical constants

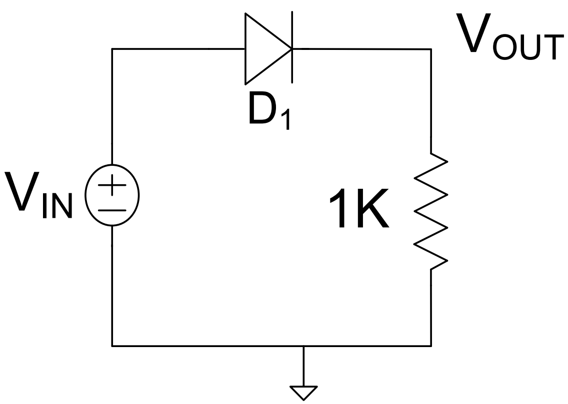
Diode Equation:
(further simplification showing more detail)

$$I(T) = \begin{cases} \left(J_{SX} \left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V}{V_t}} & V>0 \\ 0 & V<0 \end{cases}$$

Typical values for key parameters: $J_{SX}=0.5A/\mu^2$, $V_{G0}=1.17V$, m=2.3

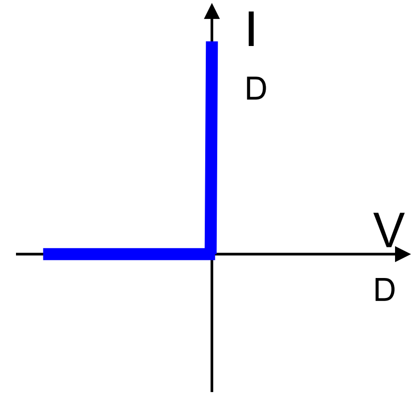
This simplification is a piecewise model !

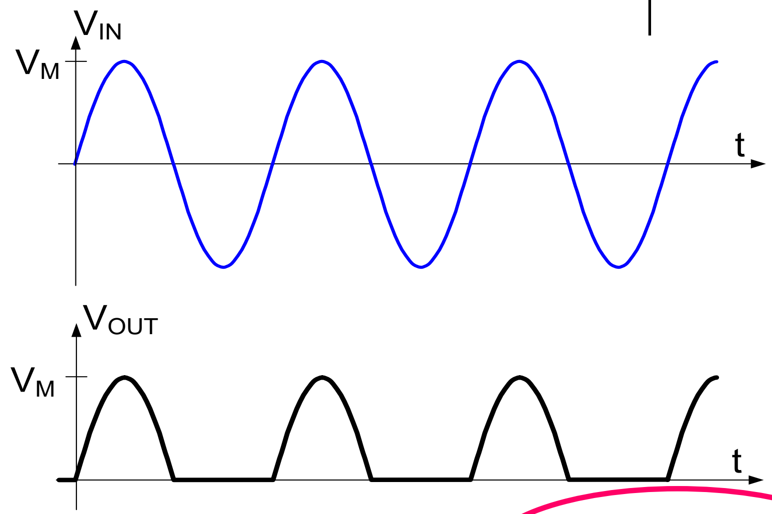
Rectifier Application:



$V_{IN}=V_M\sin\omega t$

Simple Diode Model:



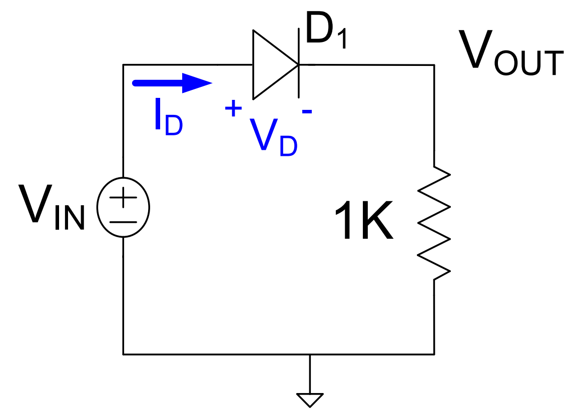


Analysis based upon "passing current" in one direction and " blocking current" in the other direction

What principle was used in this analysis?

Was this analysis rigorous ?

Diode Equation (even simplification) unwieldly to work with analytically. Why?



V_{IN}=5V

World's simplest diode circuit

Determine V_{OUT}

Assume forward bias , simplified diode equation model

$$5 = V_D + V_{OUT}$$
$$V_{OUT} = I_D \bullet 1K$$
$$I_D = I_S e^{\frac{V_D}{nV_t}}$$

3 independent equations and 3 unknowns

$$V_{OUT} = I_S e^{\frac{5-V_{OUT}}{nV_t}} \bullet 1K$$

V_{OUT}=?

- Can obtain V_{OUT} from this equation but explicit expression does not exist for V_{OUT} !
- Previous analysis based upon "passing" and "blocking" currents was not rigorous !!

I-V characteristics of pn junction

(signal or rectifier diode)

Diode Equation

$$I_D = I_S\left(e^{\frac{V_d}{nV_t}} - 1\right)$$

I_S often in the 10fA to 100fA range

I_S proportional to junction area

V_t is about 26mV at room temp

Simplification of Diode Equation:

$$I_D = \begin{cases} I_S e^{\frac{V_D}{nV_T}} & V > 0 \\ -I_S & V < 0 \end{cases}$$

How much error is introduced using the simplification for V_d > 0.5V ? (assume n=1)

$$\varepsilon = \frac{I_S\left(e^{\frac{V_d}{V_t}} - 1\right) - I_S e^{\frac{V_d}{V_t}}}{I_S\left(e^{\frac{V_d}{V_t}} - 1\right)}$$

$$\varepsilon < \frac{1}{e^{\frac{0.5}{.026}}} = 4.4 \bullet 10^{-9}$$

How much error is introduced using the simplification for V_d < - 0.5V?

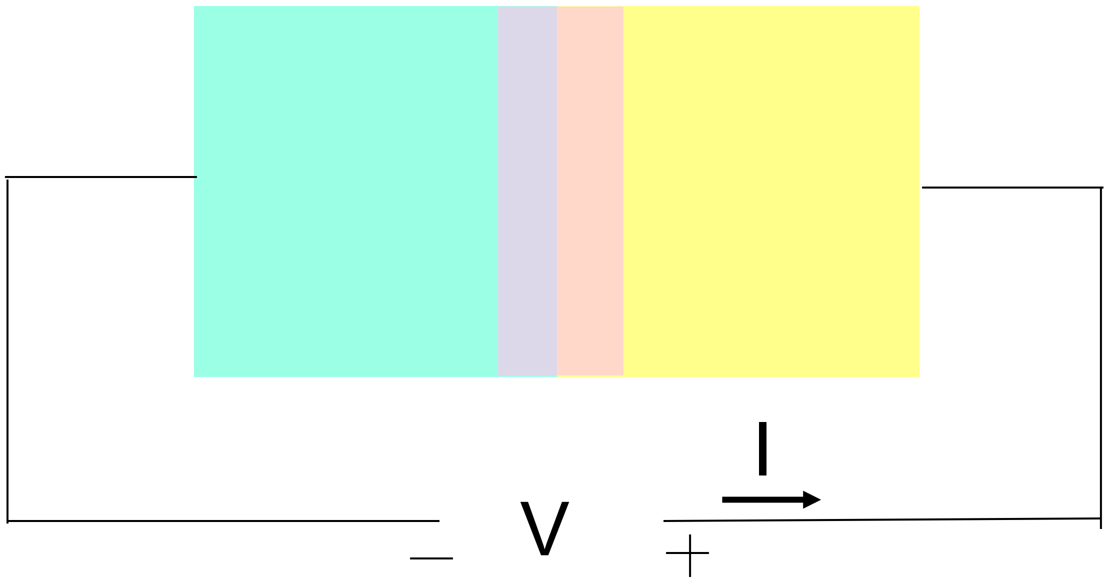
$$\varepsilon < e^{\frac{-0.5}{.026}} = 4.4 \bullet 10^{-9}$$

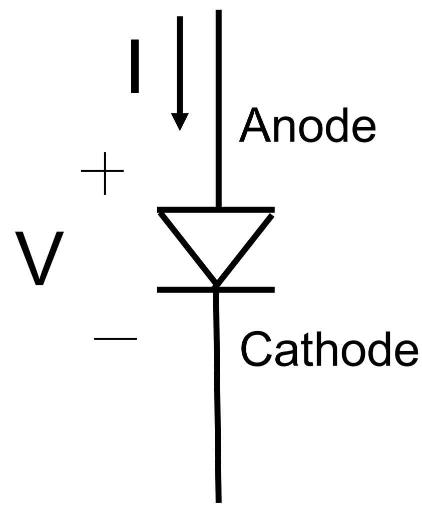
Simplification almost never introduces any significant error

Will you impress your colleagues or your boss if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?

Will your colleagues or your boss be unimpressed if you use the more exact diode equation when $V_d < -0.5V$ or $V_d > +0.5V$?

pn Junctions





"Diode Equation":

(good enough for most applications when ideal diode model is inadequate)

$$I = \begin{cases} J_S A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

Note: $I_S = J_s A$

J_S= Sat Current Density (in the 1aA/u^2 to 1fA/u^2 range)

A= Junction Cross Section Area

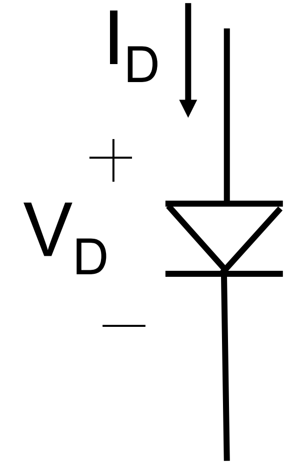
V_T=kT/q (k/q=1.381x10^{-23}V•C/°K/1.6x10^{-19}C=8.62x10^{-5}V/°K)

n is approximately 1

I_S highly temperature dependent

Example: Consider diode operating under forward bias

$$I_D(T) = \left(J_{SX}\left[T^m e^{\frac{-V_{G0}}{V_t}} \right] \right) A e^{\frac{V_D}{V_t}}$$



What percent change in I_S will occur for a 1°C change in temperature at room temperature?

$$\frac{\Delta I_S}{I_S} = \frac{\left(J_{SX}\left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) A - \left(J_{SX}\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A}{\left(J_{SX}\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right) A} = \frac{\left(\left[T_{T_2}^m e^{\frac{-V_{G0}}{V_t(T_2)}} \right] \right) - \left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}{\left(\left[T_{T_1}^m e^{\frac{-V_{G0}}{V_t(T_1)}} \right] \right)}$$

$$\frac{\Delta I_S}{I_S} = \frac{\left(1.240x10^{-15}\right) - \left(1.025x10^{-15}\right)}{\left(1.025x10^{-15}\right)} 100\% = 21\%$$

- Attempts to measure I_S in our laboratories can result in large errors !
- Most circuits whose performance depends upon precise value for I_S are not practical

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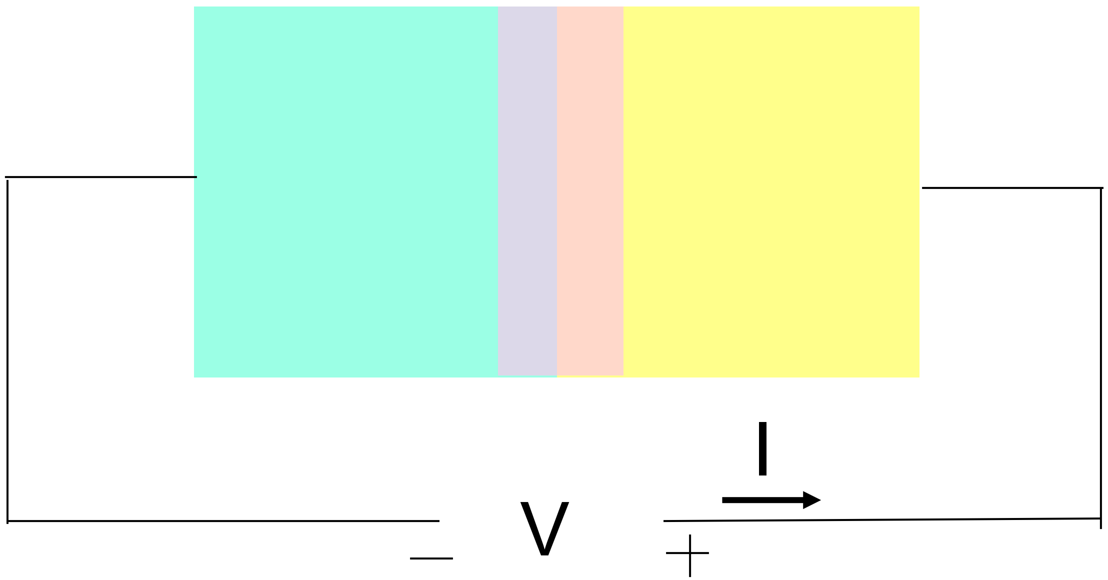
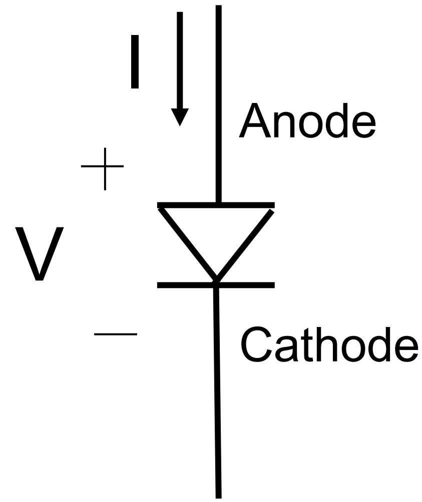


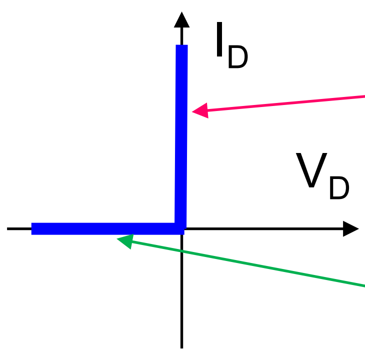


Diode Equation:
(good enough for most applications)

$$I = \begin{cases} J_S A e^{\frac{V}{nV_T}} & V > 0 \\ 0 & V < 0 \end{cases}$$

$I_S = J_s A$

Simple Diode Model:

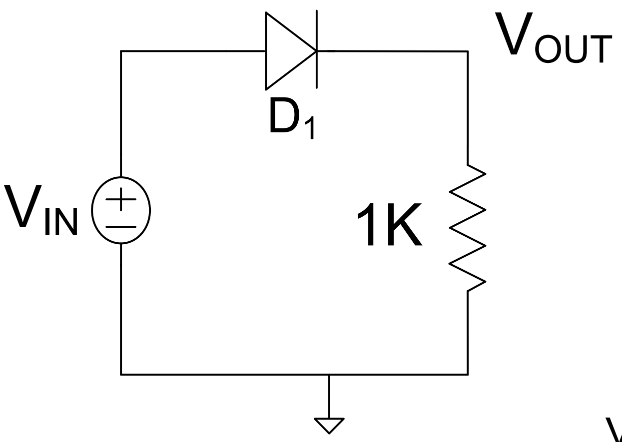


Often termed the “conducting” or “ON” state

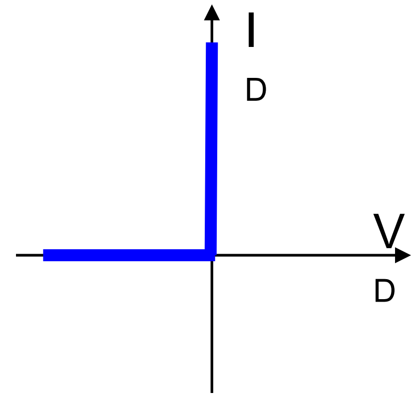
Often termed the “nonconducting” or “OFF” state

What basic circuit analysis principles were used to analyze this circuit?

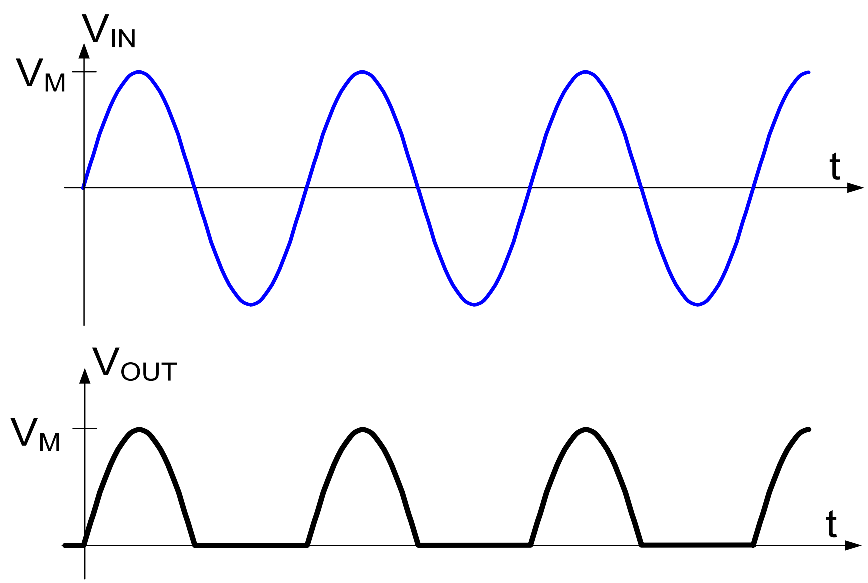
Rectifier Application:



Simple Diode Model:

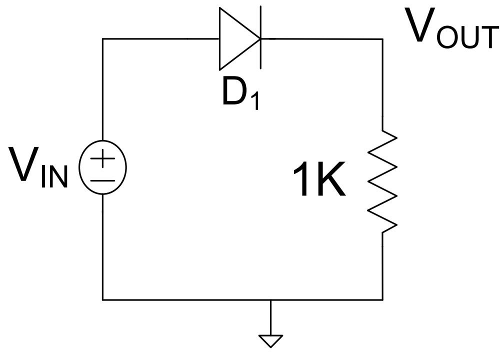


$V_{IN}=V_M \sin\omega t$



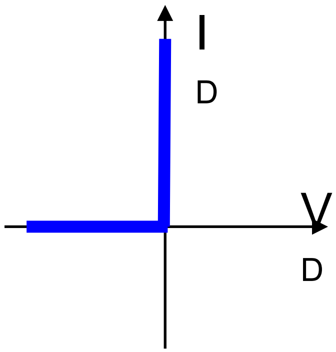
Analysis based upon "passing current" in one direction and " blocking current" in the other direction

Rectifier Application:



$V_{IN}=V_M \sin\omega t$

Simple Diode Model:

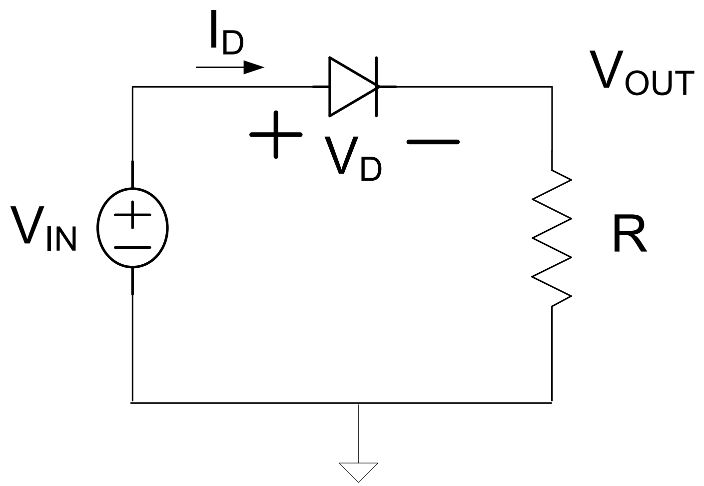


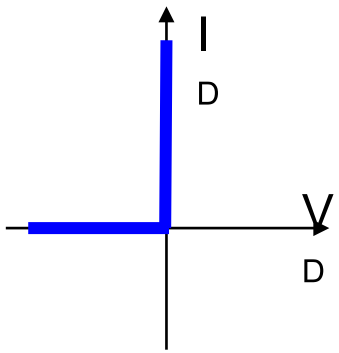
Analysis based upon "passing current" in one direction and " blocking current" in the other direction

Was the previous analysis rigorous?

Is use of simple diode model justifiable?

Consider again the basic rectifier circuit

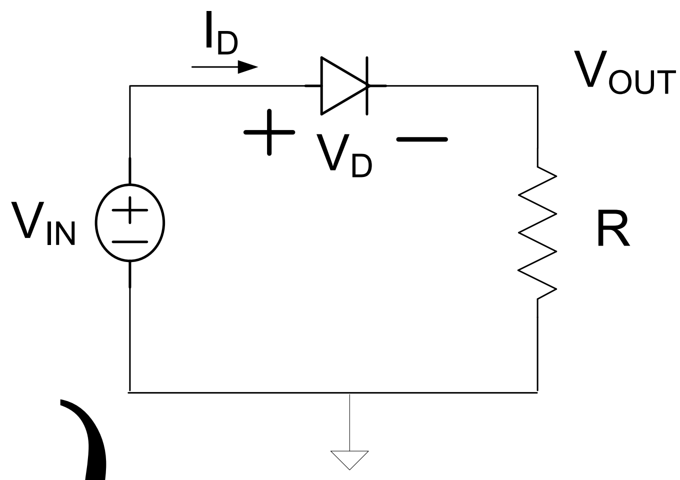




- Previously considered sinusoidal excitation
- Previously gave "qualitative" analysis
- **Rigorous analysis method is essential**

$$V_{OUT} = \ ?$$

Consider again the basic rectifier circuit



$$\left.\begin{aligned} V_{IN} &= V_D + I_D R \\ V_{OUT} &= I_D R \\ I_D &= I_S\left(e^{\frac{V_D}{V_t}} - 1\right) \end{aligned}\right\}$$

$$V_{OUT} = I_S R\left(e^{\frac{V_{IN}-V_{OUT}}{V_t}} - 1\right)$$

This analysis is rigorous (using only KVL and device models)

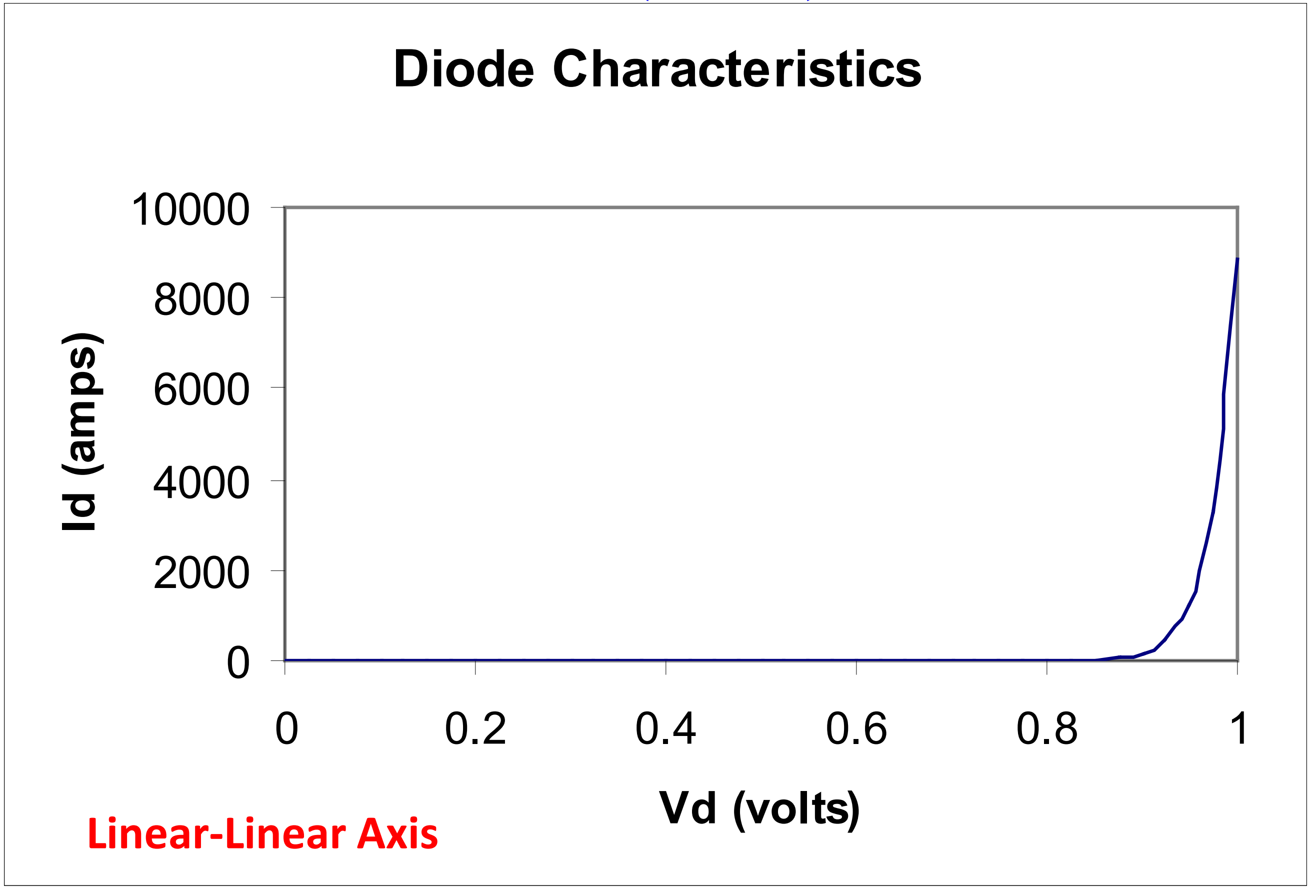
Even the simplest diode circuit does not have a closed-form explicit solution when diode equation is used to model the diode !!

Due to the nonlinear nature of the diode equation

Simplifications of diode model are essential if analytical results are to be obtained !

Lets study the diode equation a little further

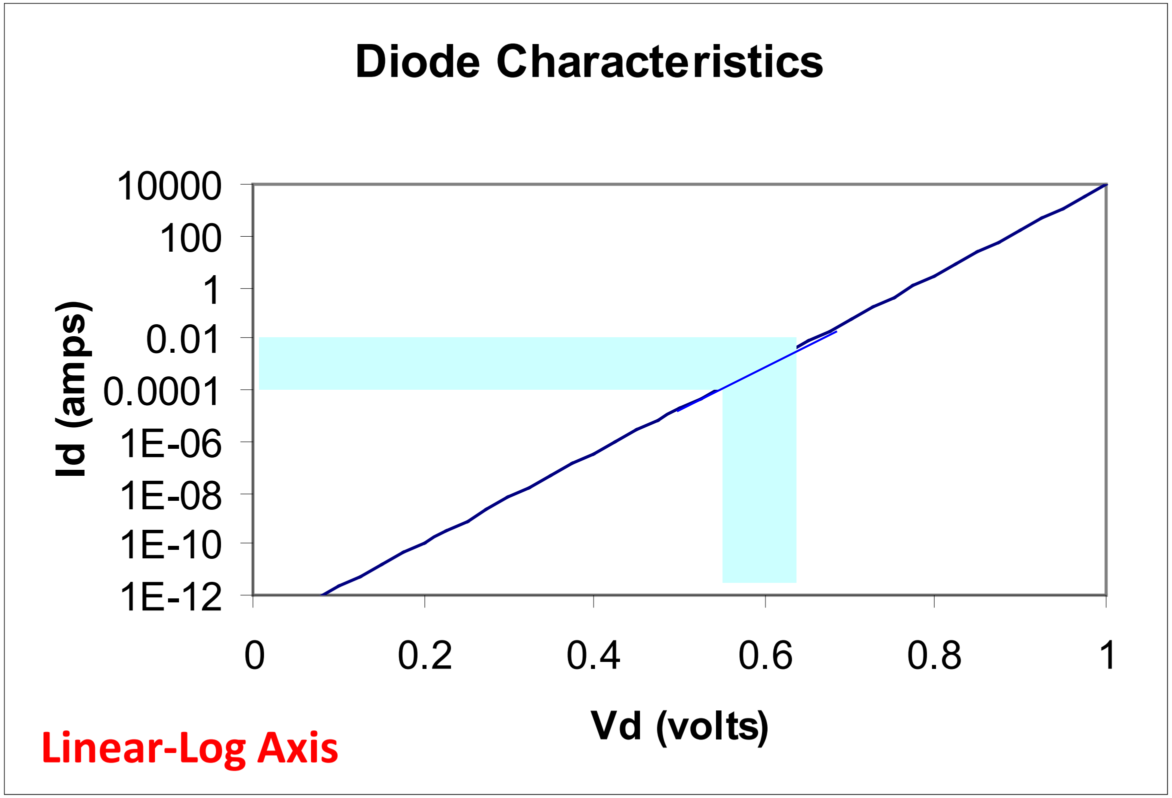
$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$



Power Dissipation Becomes Destructive if Vd > 0.85V (actually less)

Lets study the diode equation a little further

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

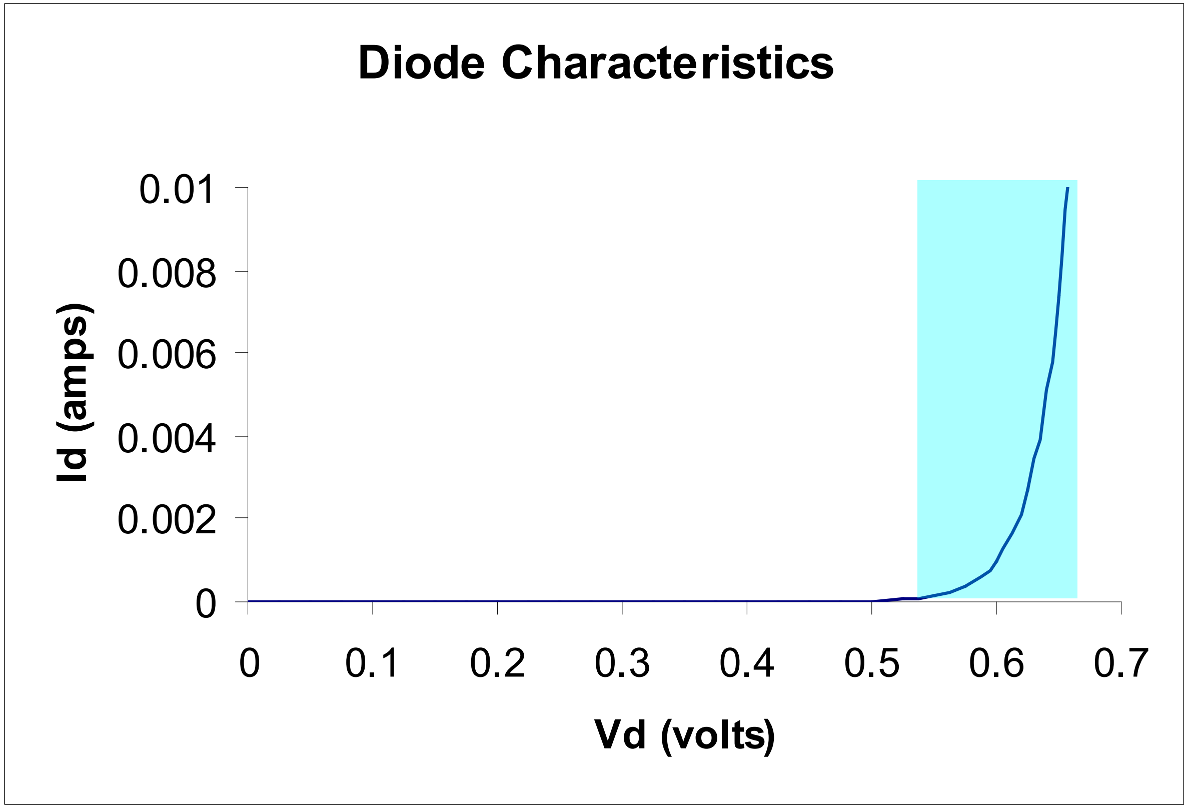


For two decades of current change, Vd is close to 0.6V

This is the most useful conducting current range for many applications

Lets study the diode equation a little further

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$



For two decades of current change, Vd is close to 0.6V

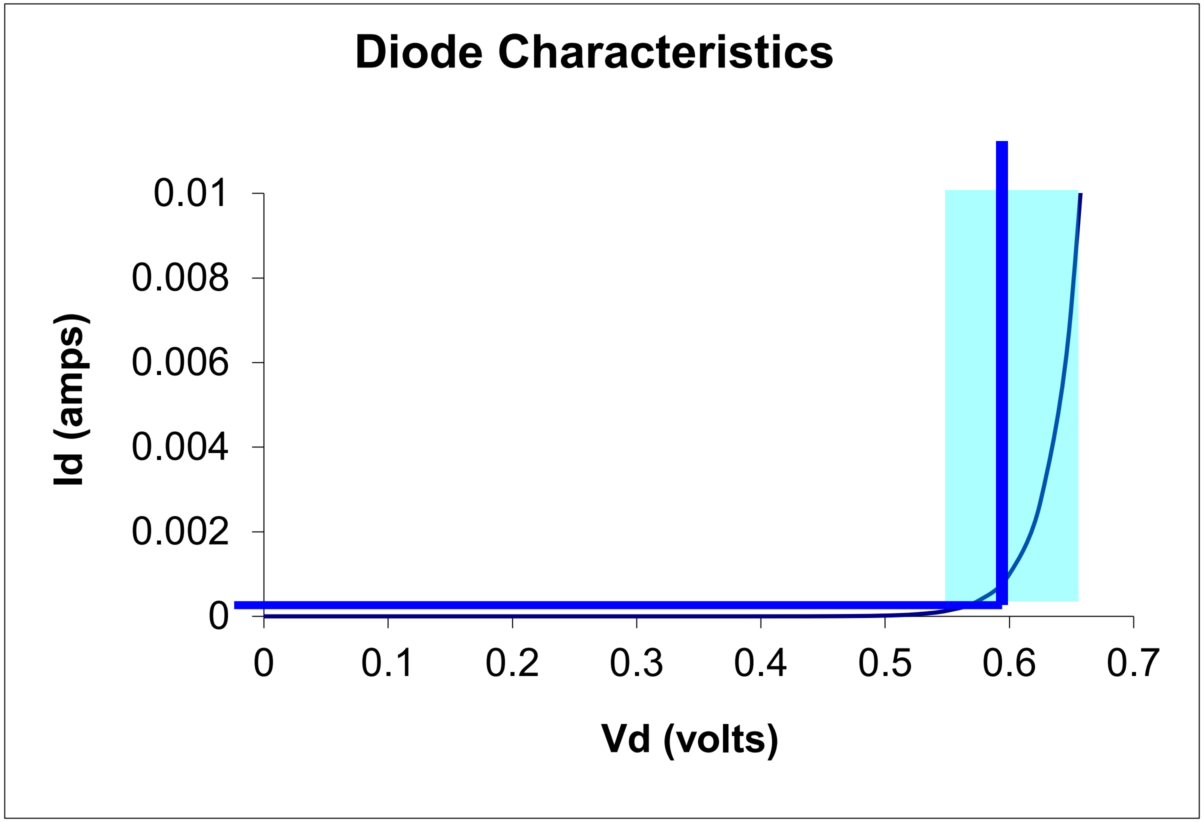
This is the most useful current range when conducting for many applications

Lets study the diode equation a little further

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

$$I_d = 0 \qquad V_d < 0.6\ V$$

$$V_d = 0.6\ V \qquad I_d > 0$$



Widely Used **Piecewise Linear** Model

Which simplified model is better?

Both are about the same !

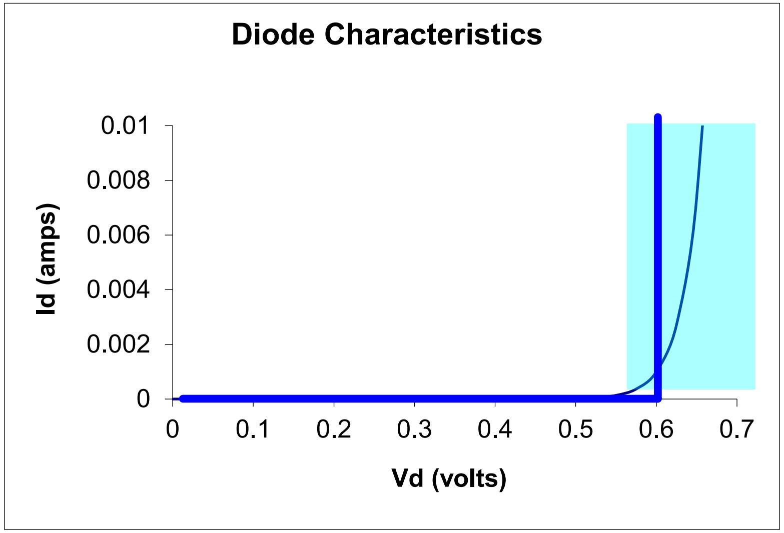
$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

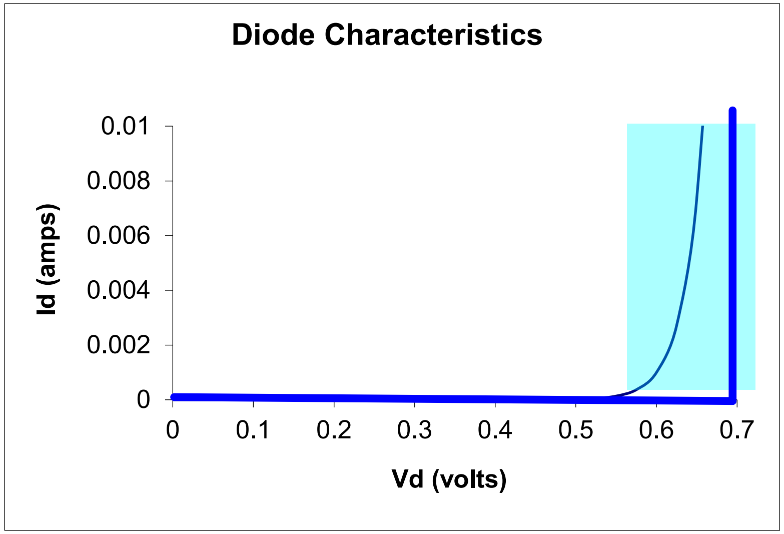
$$I_d = 0 \qquad V_d < 0.6\ V$$

$$V_d = 0.6\ V \qquad I_d > 0$$

$$I_d = 0 \qquad V_d < 0.7\ V$$

$$V_d = 0.7\ V \qquad I_d > 0$$





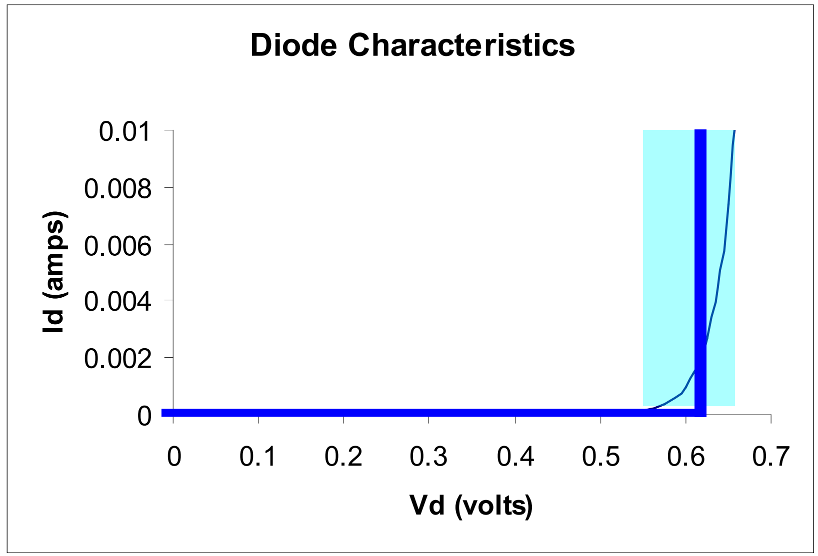
Widely Used **<u>Piecewise Linear</u>** Model

Lets study the diode equation a little further

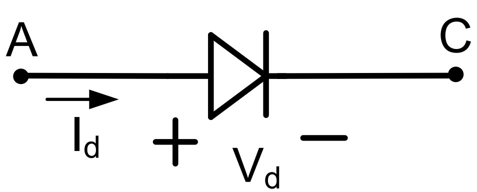
$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

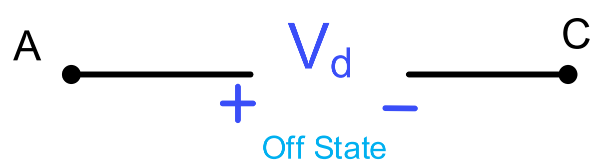
$$I_d = 0 \qquad V_d < 0.6\ \mathrm{V}$$

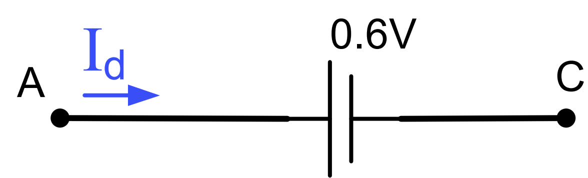
$$V_d = 0.6\ \mathrm{V} \qquad I_d > 0$$



Equivalent Circuit

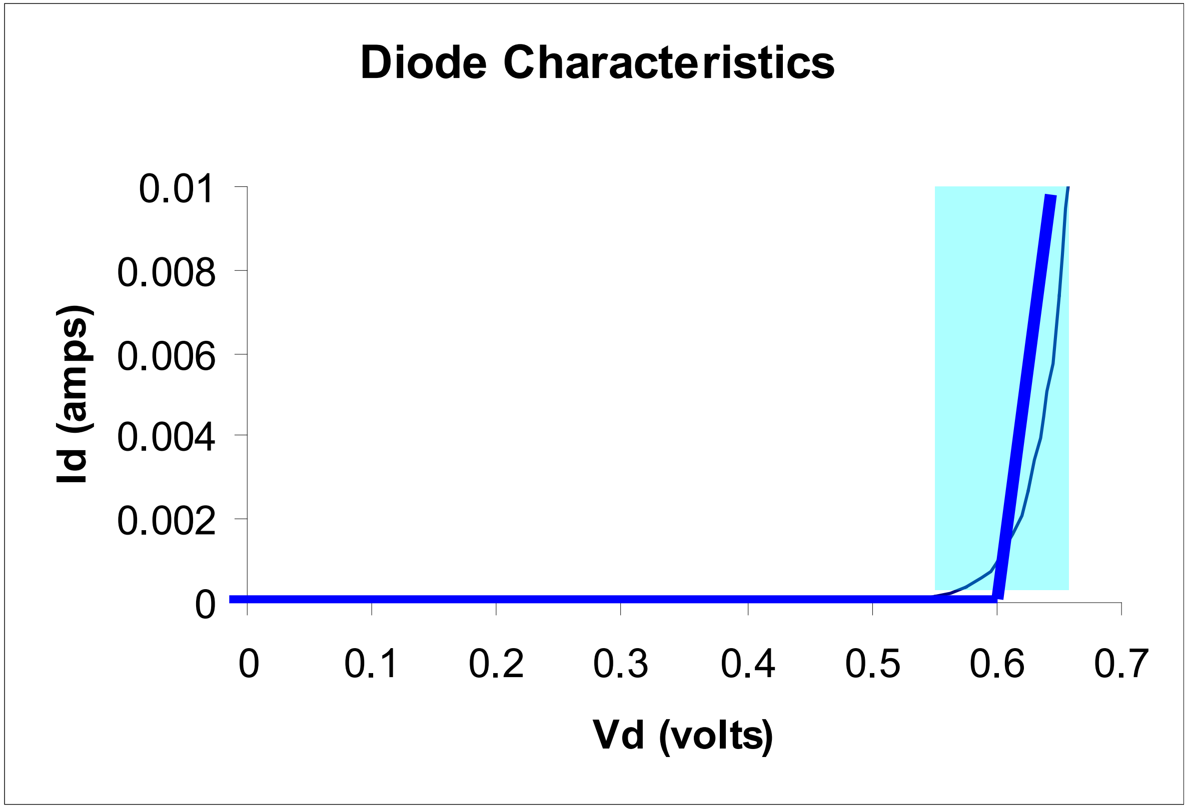






Lets study the diode equation a little further

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$



Better model in “ON” state though often not needed

Includes Diode “ON” resistance

Lets study the diode equation a little further

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

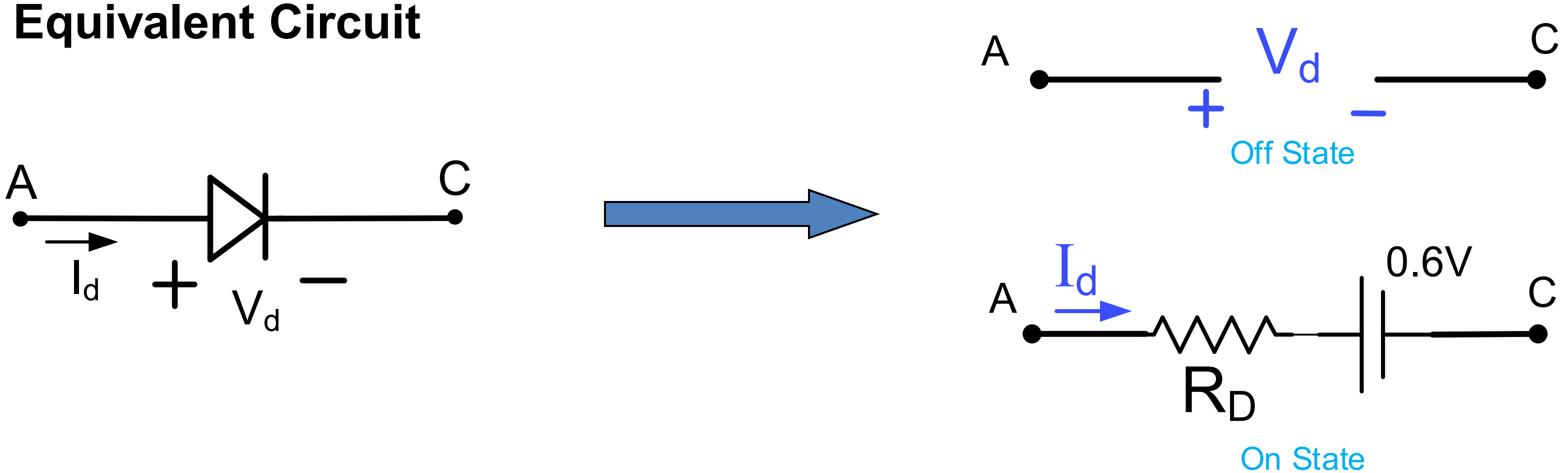
Piecewise Linear Model with Diode Resistance

$$I_d = 0 \qquad \text{if } V_d < 0.6V$$

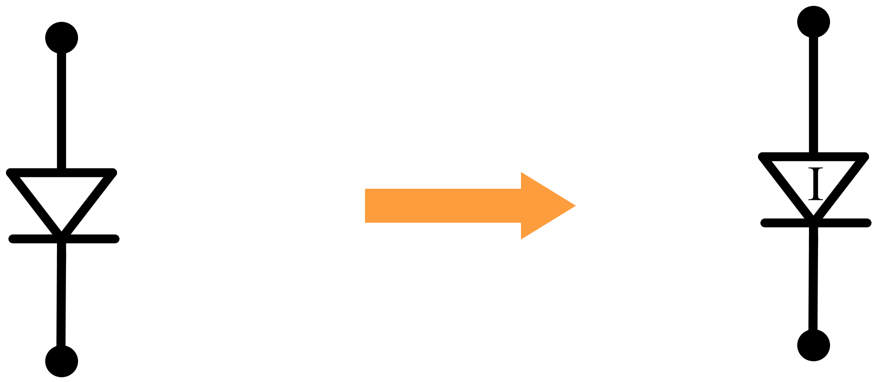
$$V_d = 0.6\ V + I_d R_D \qquad \text{if } I_d > 0$$

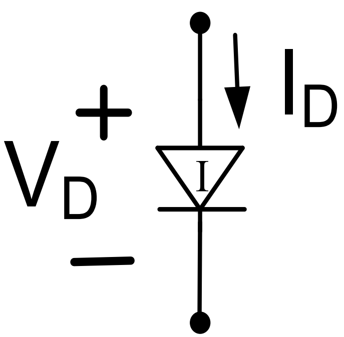
(R_D is rather small: often in the 20Ω to 100Ω range):

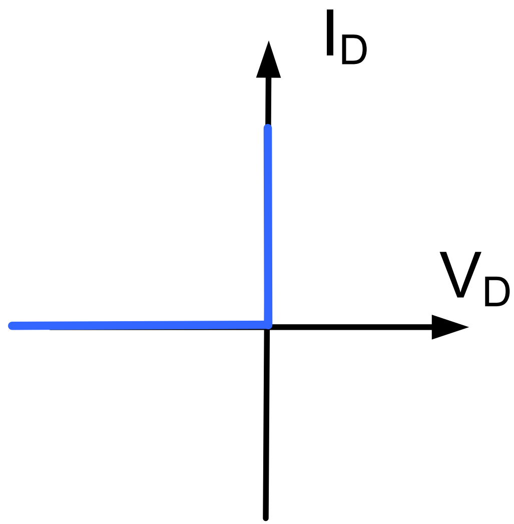
Equivalent Circuit



The Ideal Diode



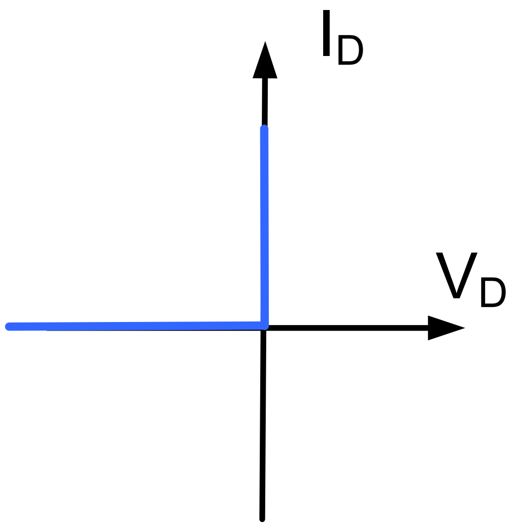




$$I_D = 0 \quad \text{if} \quad V_D \leq 0$$

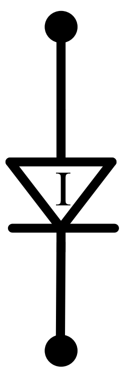
$$V_D = 0 \quad \text{if} \quad I_D > 0$$

The Ideal Diode

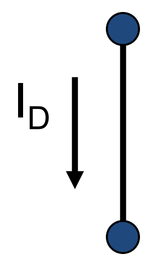


$$I_D = 0 \quad \text{if} \quad V_D \leq 0$$ “OFF”

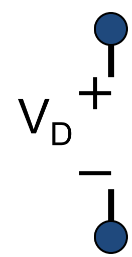
$$V_D = 0 \quad \text{if} \quad I_D > 0$$ “ON”



“ON”



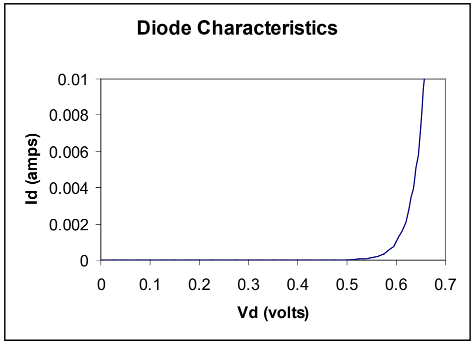
“OFF”

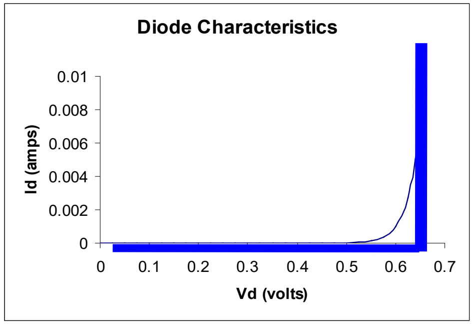


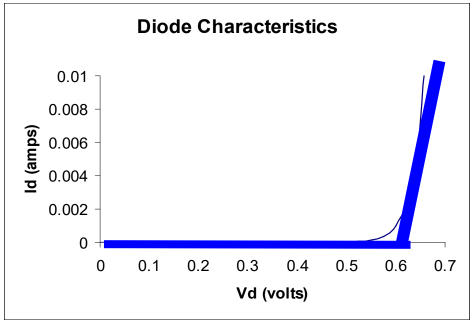
Valid for $I_D > 0$ $V_D \leq 0$

Diode Models

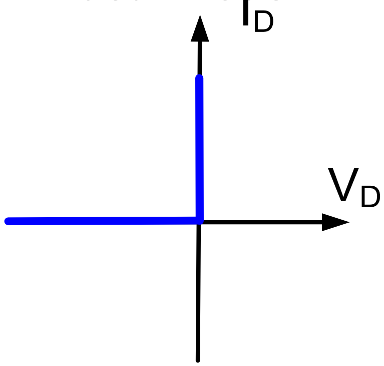
Diode Equation (4 variants)







Ideal Model



Which model should be used?

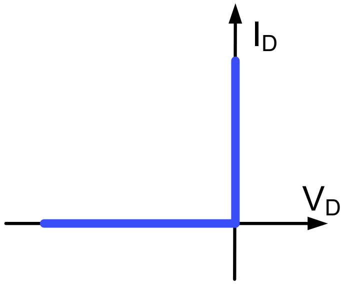
The simplest model that will give acceptable results in the analysis of a circuit

Diode Model Summary

Piecewise Linear Models

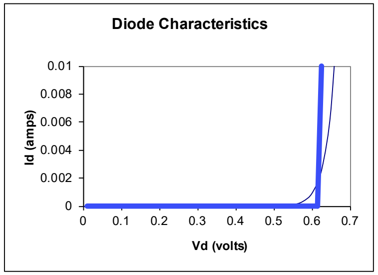
$I_d = 0$ if $V_d < 0$

$V_d = 0$ if $I_d > 0$



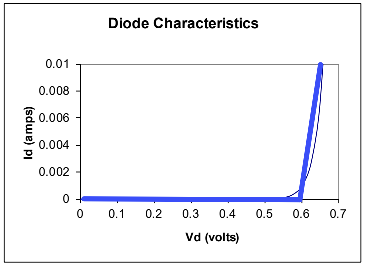
$I_d = 0$ if $V_d < 0.6V$

$V_d = 0.6V$ if $I_d > 0$



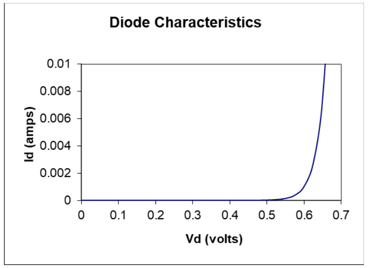
$I_d = 0$ if $V_d < 0.6$

$V_d = 0.6 + I_d R_d$ if $I_d > 0$



Diode Equation (or variants discussed)

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$



Diode Model Summary

Piecewise Linear Models

$I_d = 0$ if $V_d < 0$

$V_d = 0$ if $I_d > 0$

$I_d = 0$ if $V_d < 0.6V$

$V_d = 0.6V$ if $I_d > 0$

$I_d = 0$ if $V_d < 0.6$

$V_d = 0.6 + I_d R_d$ if $I_d > 0$

Diode Equation (or variants discussed)

$$I_d = I_S\left(e^{\frac{V_d}{V_t}} - 1\right)$$

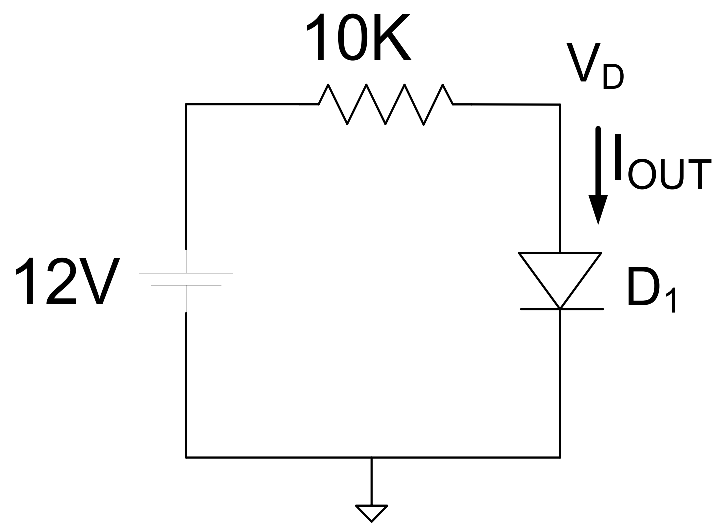
When is the ideal model adequate?

When it doesn't make much difference whether V_d=0V or V_d=0.6V

When is the second piecewise-linear model adequate?

When it doesn't make much difference whether V_d=0.6V or V_d=0.7V

Example: Determine I_{OUT} for the following circuit



Solution:

If the diode equation model is used will obtain:

$$
\left.\begin{aligned}
12 &= I_{OUT} \bullet 10K + V_D \\
I_{OUT} &= I_S\left(e^{\frac{V_D}{V_t}} - 1\right)
\end{aligned}\right\} \Longrightarrow \quad I_{OUT} = I_S\left(e^{\frac{-I_{OUT}\bullet 10K}{V_t}} e^{\frac{12}{V_t}} - 1\right)
$$

As in previous example, a closed-form explicit expression for I_{OUT} does not exist

Will now establish rigorous approach for solving this (and other) nonlinear circuit (with model uncertainty and piecewise models) with piecewise models and obtaining a practical solution !

Devices in Semiconductor Processes

- Resistors
- Diodes
- Capacitors
- MOSFETs

→ Side Track!
Analysis of Nonlinear Circuits

Analysis of Nonlinear Circuits

(Circuits with one or more nonlinear devices)

What analysis tools or methods can be used?

KCL ?

KVL?

~~Superposition?~~

~~Voltage Divider ?~~

~~Current Divider?~~

~~Thevenin and Norton Equivalent Circuits?~~

Nodal Analysis ?

Mesh Analysis ?

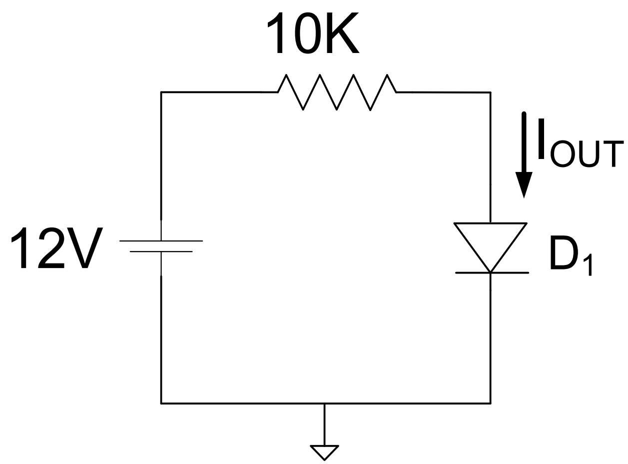
Two-Port Subcircuits ?

~~Passing Current ?~~

~~Blocking Current ?~~

- How are piecewise models accommodated?
- Will address the issue of how to rigorously analyze nonlinear circuits with piecewise models later

Example: Determine I_{OUT} for the following circuit



$$I_{OUT} = I_S\left(e^{\frac{-I_{OUT}\bullet 10K}{V_t}} e^{\frac{12}{V_t}} - 1\right)$$

- Results are accurate
- Analysis was tedious (and if slightly more complicated circuit even single implicit expression for output is often not attainable)
- Difficult to interpret results with implicit solution

Alternate Solution Strategy:

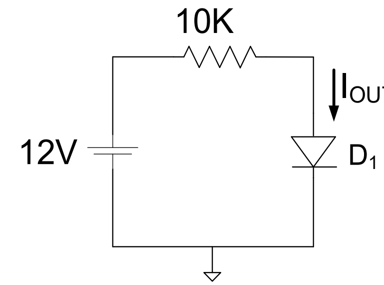
1. Assume PWL model with V_D=0.6V, R_D=0
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2 (verify the "if" condition in model)

Select Model

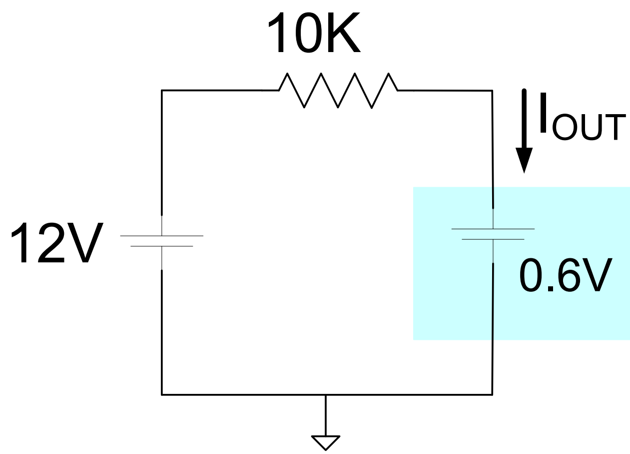
5. Assume PWL with V_D=0.7V
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6 (verify the "if" condition in model)
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Validate Model

Alternate Solution:



1. Assume PWL model with V_D=0.6V, R_D=0, IS=10FA
2. Guess state of diode (ON)



3. Analyze circuit with model

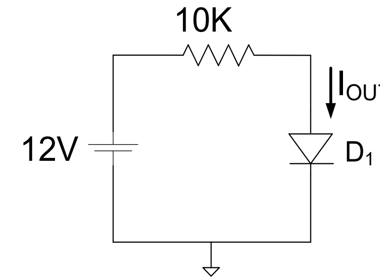
$$I_{OUT}=\frac{12V-0.6V}{10K}=1.14mA$$

4. Validate state of guess in step 2

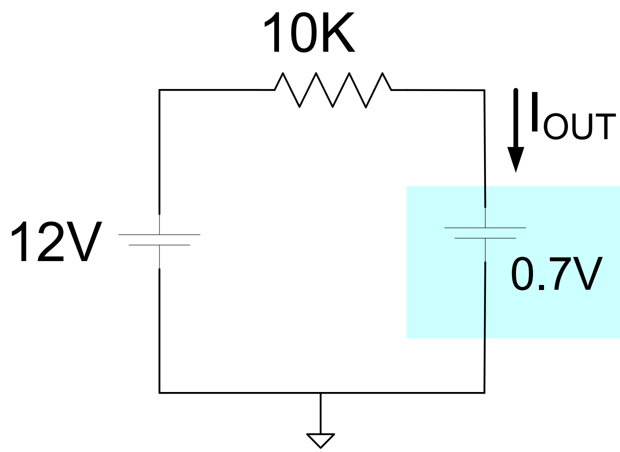
To validate state, must show $I_D>0$

$$I_D=I_{OUT}=1.14mA>0$$

Alternate Solution:



5. Assume PWL model with V_D=0.7V, R_D=0, IS=10FA
6. Guess state of diode (ON)



7. Analyze circuit with model

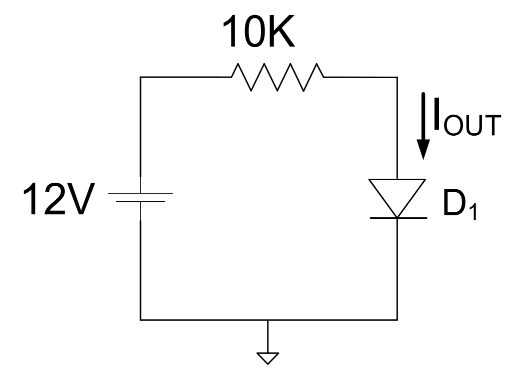
$$I_{OUT} = \frac{12V-0.7V}{10K} = 1.13mA$$

8. Validate state of guess in step 6

To validate state, must show I_D>0

$$I_D = I_{OUT} = 1.13mA > 0$$

Alternate Solution:



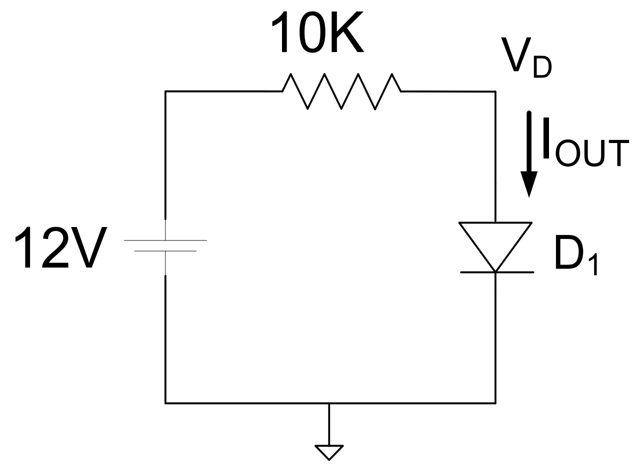
9. Show difference between results using these two models is small

$I_{OUT}=1.14mA$ and $I_{OUT}=1.13$ mA are close

Thus, can conclude $I_{OUT} \cong 1.14mA$

Example:

Determine I_{OUT} for the following circuit



How do the two solutions compare?

With diode equation model for IS=10fA :

$$I_{OUT} = I_S\left(e^{\frac{-I_{OUT}\bullet 10K}{V_t}} e^{\frac{12}{V_t}} - 1\right) \Rightarrow I_{OUT}=1.134mA$$

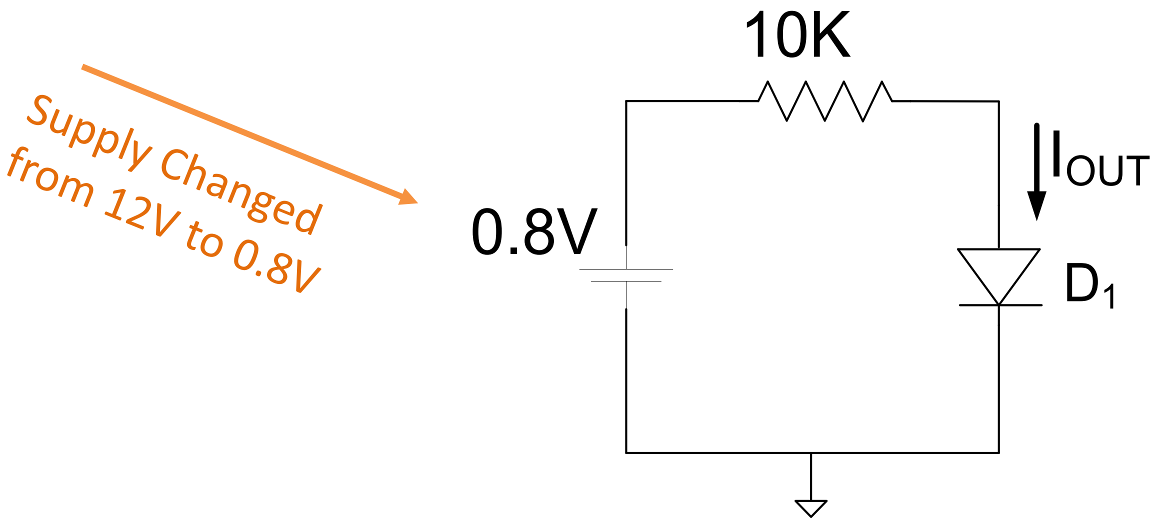
With PWL model:

$$I_{OUT} \cong 1.14mA$$

What was the major reason the PWL model simplified the analysis?

Piecewise Linear Model

Example: Determine I_{OUT} for the following circuit



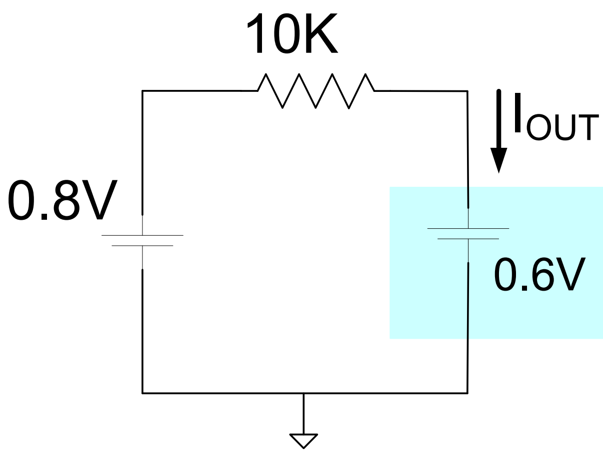
Solution:

Strategy:

1. Assume PWL model with V_D=0.6V, R_D=0
2. Guess state of diode (ON)
3. Analyze circuit with model
4. Validate state of guess in step 2
5. Assume PWL with V_D=0.7V
6. Guess state of diode (ON)
7. Analyze circuit with model
8. Validate state of guess in step 6
9. Show difference between results using these two models is small
10. If difference is not small, must use a different model

Solution:

1. Assume PWL model with V_D=0.6V, R_D=0
2. Guess state of diode (ON)



3. Analyze circuit with model

$$I_{OUT} = \frac{0.8 - 0.6V}{10K} = 20\mu A$$

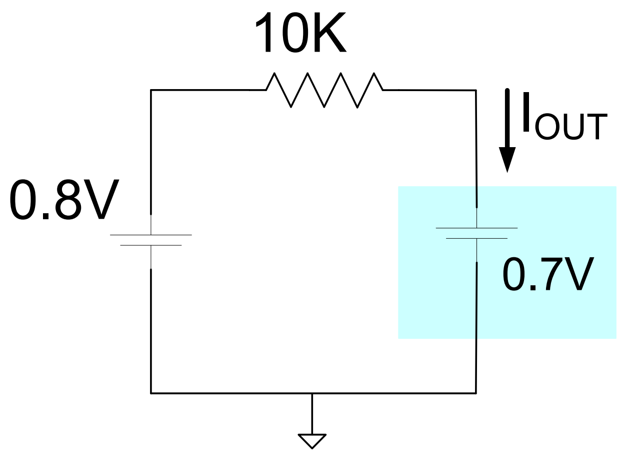
4. Validate state of guess in step 2

To validate state, must show I_D>0

$$I_D = I_{OUT} = 20\mu A > 0$$

Solution:

5. Assume PWL model with V_D=0.7V, R_D=0
6. Guess state of diode (ON)



7. Analyze circuit with model

$$I_{OUT} = \frac{0.8V-0.7V}{10K} = 10\mu A$$

8. Validate state of guess in step 6

To validate state, must show I_D>0

$$I_D = I_{OUT} = 10\mu A > 0$$

Solution:

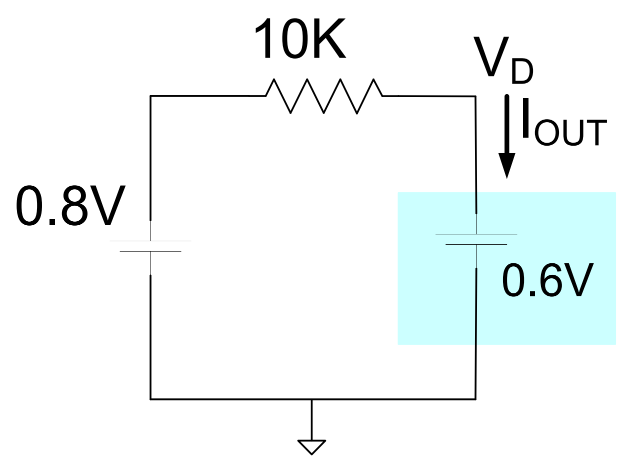
9. Show difference between results using these two models is small

$I_{OUT}=10\mu A$ and $I_{OUT}=20\mu A$ are not close

10. If difference is not small, must use a different model

Thus must use diode equation to model the device

$$\left.\begin{aligned} I_{OUT} &= \frac{0.8-V_D}{10K} \\ I_{OUT} &= I_S e^{\frac{V_D}{V_t}} \end{aligned}\right\}$$



Solve simultaneously, assume V_t=25mV, I_S=1fA

Solving these two equations by iteration, obtain V_D= 0.6148V and I_{OUT}=18.60μA

Use of Piecewise Models for Nonlinear Devices when Analyzing Electronic Circuits

Process:

1. Guess state of the device
2. Analyze circuit
3. Verify State
4. Repeat steps 1 to 3 if verification fails
5. Verify model (if necessary)

Observations:

- Analysis generally simplified dramatically (particularly if piecewise model is linear)
- Approach applicable to wide variety of nonlinear devices
- Usually much faster than solving the nonlinear circuit directly
- Wrong guesses in the state of the device do not compromise solution (verification will fail)
- Helps to guess right the first time
- Detailed model is often not necessary with most nonlinear devices
- Particularly useful if piecewise model is PWL (but not necessary)
- Closed-form solutions (attainable with PWL models) give insight into performance of circuit
- For practical circuits, the simplified approach with piecewise models usually applies

Key Concept For Analyzing Circuits with Nonlinear Devices

Stay Safe and Stay Healthy !

End of Lecture 15